# Report on Reflectance in Dielectric Structures

April 2025

## 1 Introduction and Overview

This report addresses three C programs developed to calculate the reflectance of dielectric structures:

1. a single-layer dielectric coating,
2. a dielectric bilayer, and
3. a multilayer dielectric coating.

The underlying method is the transfer-matrix formalism for normally incident light. The original report already contained the computational workflow and physical interpretation, but it referred externally to a problem sheet for part of the theoretical setup. The present version makes only minimal changes to the structure and prose of that report, while incorporating the required mathematical definitions directly so that the document is self-contained.

### 1.1 Theoretical underpinning

For a dielectric layer, the reflectance is

$$R = |r|^2 = rr^*, \tag{1}$$

where $r$ is the complex reflectivity. The transfer-matrix method expresses the electric and magnetic fields on one side of a layer in terms of those on the other side. For a single layer $L$,

$$\begin{pmatrix} E_1 \\ H_1 \end{pmatrix} = M_L \begin{pmatrix} E_2 \\ H_2 \end{pmatrix}. \tag{2}$$

For normal incidence, the transfer matrix is

$$M_L = \begin{pmatrix} \cos(k_0 h_L) & \dfrac{i \sin(k_0 h_L)}{Y_L} \\ iY_L \sin(k_0 h_L) & \cos(k_0 h_L) \end{pmatrix}, \tag{3}$$

with

$$k_0 = \frac{2\pi}{\lambda}, \qquad h_L = n_L d_L, \qquad Y_L = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_L. \tag{4}$$

Here $\lambda$ is the wavelength in vacuum, $n_L$ is the refractive index of the layer, $d_L$ is its physical thickness, $h_L$ is its optical thickness, and $Y_L$ is the optical admittance.

For a second layer $H$ with refractive index $n_H$ and thickness $d_H$, the corresponding matrix is

$$M_H = \begin{pmatrix} \cos(k_0 h_H) & \dfrac{i\sin(k_0 h_H)}{Y_H} \\ iY_H \sin(k_0 h_H) & \cos(k_0 h_H) \end{pmatrix}, \tag{5}$$

where

$$h_H = n_H d_H, \qquad Y_H = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_H. \tag{6}$$

For a stack of $N$ layers, the total transfer matrix is the ordered product

$$M = M_1 M_2 M_3 \cdots M_N = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}. \tag{7}$$

The complex reflectivity is then

$$r = \frac{Y_0 M_{11} + Y_0 Y_S M_{12} - M_{21} - Y_S M_{22}}{Y_0 M_{11} + Y_0 Y_S M_{12} + M_{21} + Y_S M_{22}}, \tag{8}$$

where

$$Y_0 = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_0, \qquad Y_S = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, n_S. \tag{9}$$

In the present calculations, the entry medium is air so $n_0 = 1.0$, and the substrate is glass with $n_S = 1.5$.

To calculate the reflectance of a single dielectric layer at specific wavelengths (450 nm, 550 nm, 650 nm), the total matrix is evaluated using equation (3), then the reflectivity is obtained from equation (8), and finally equation (1) gives the reflectance. The same steps apply to bilayer and multilayer structures, except that the total matrix is constructed by multiplying together the transfer matrices of all constituent layers.

## 1.2 Boilerplate reflectance library

From the end of the last section, three highly generalisable ideas emerge: the layer transfer matrix, the reflectivity formula, and the spectrum sweep over wavelength. Since these operations are common to the single-layer, bilayer, and multilayer programs, they are factored into a small reflectance library. In the original submission, the core functions were also repeated explicitly in the source listings so that each program remained standalone.

Listing 1: Single-layer transfer matrix routine.

```
#define SQRT_EPS0_OVER_MU0 0.002654f


void create_transfer_matrix(fcomplex mat[2][2], float n, float k0, float d) {
```

```
    const float kh = k0 * n * d;
    const float Y = SQRT_EPS0_OVER_MU0 * n;
    const fcomplex sin_term = complex(0.0f, sinf(kh));

    mat[0][0] = complex(cosf(kh), 0.0f);
    mat[0][1] = cdiv(sin_term, complex(Y, 0.0f));
    mat[1][0] = rcmult(Y, sin_term);
    mat[1][1] = complex(cosf(kh), 0.0f);
}
```

This routine implements equations (3) and (5) directly. The parameter $kh = k_0 n d$ is the phase thickness, and the admittance is $Y = \sqrt{\varepsilon_0/\mu_0}\, n$.

Listing 2: Reflectance evaluation routine.

```
float evaluate_R(fcomplex M[2][2], float n0, float ns) {
    const fcomplex Y0 = complex(SQRT_EPS0_OVER_MU0 * n0, 0.0f);
    const fcomplex Ys = complex(SQRT_EPS0_OVER_MU0 * ns, 0.0f);

    fcomplex term1 = cmult(Y0, M[0][0]);
    fcomplex term2 = cmult(cmult(Y0, Ys), M[0][1]);
    fcomplex term3 = csub(complex(0,0), M[1][0]);
    fcomplex term4 = csub(complex(0,0), cmult(Ys, M[1][1]));
    fcomplex numerator = cadd(cadd(term1, term2), cadd(term3, term4));

    fcomplex denominator = cadd(cadd(term1, term2),
                                cadd(M[1][0], cmult(Ys, M[1][1])));

    fcomplex r = cdiv(numerator, denominator);
    return cabs(r) * cabs(r);
}
```

The numerator and denominator are evaluated separately before division in order to work cleanly with the complex arithmetic library.

Listing 3: Universal wavelength sweep.

```
void spectrum(FILE *fp, TransferMatrixCB cb, int layers,
              float n0, float nL, float dL, float nH, float dH, float ns) {
    const float step_size = (layers > 1) ? 1.0f : 10.0f;
    const int total_steps = (int)((LAMBDA_MAX - LAMBDA_MIN) / step_size) + 1;

    for (int i = 0; i <= total_steps; i++) {
        float lambda = LAMBDA_MIN + i * step_size;
        if (lambda > LAMBDA_MAX) break;

        fcomplex M[2][2];
        cb(M, lambda, layers);
        float R = evaluate_R(M, n0, ns);
        fprintf(fp, "%.1f,%.6f\n", lambda, R);
```

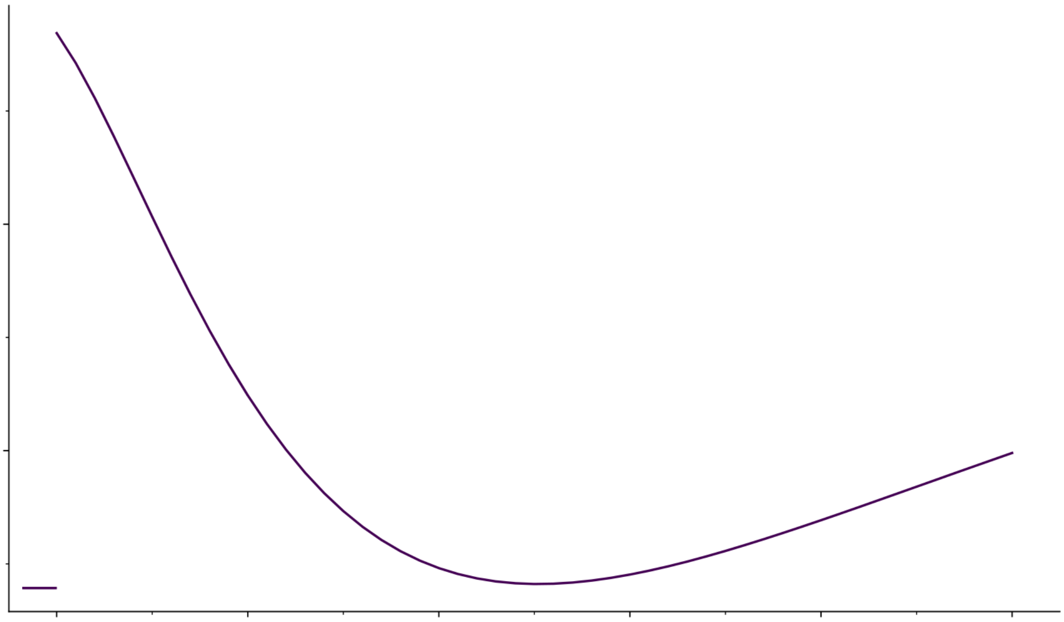

Figure 1: Reflectance spectrum of the single dielectric layer from 300 nm to 800 nm.

```
    }
}
```

This callback-based structure allows each program to define its own total transfer matrix while reusing the same reflectance and spectrum machinery.

## 2 Single-Layer Dielectric Coating

### 2.1 Model

The single-layer structure consists of a $MgF_2$-like low-index coating with refractive index $n_L = 1.38$ and thickness $d_L = 100\,\text{nm}$ deposited on glass with $n_S = 1.5$. Since there is only one layer, the total transfer matrix is simply $M = M_L$.

Listing 4: Single-layer callback.

```
void total_transfer_matrix(fcomplex M[2][2], float lambda, int _) {
    const float nL = 1.38f, dL = 100.0f;
    const float k0 = 2 * M_PI / lambda;
    create_transfer_matrix(M, nL, k0, dL);
}
```

The reflectance values for the three specific wavelengths are:

$$R(450\,\text{nm}) = 0.01733, \qquad R(550\,\text{nm}) = 0.013111, \qquad R(650\,\text{nm}) = 0.015572.$$

The reflectance spectrum for the single dielectric layer exhibits a monotonic decrease from approximately 0.035 at 300 nm to a minimum of about 0.014 near 550 nm, followed by a gradual increase to roughly 0.02 at 800 nm. This distinctive U-shaped profile results from the interference between light waves reflected at the air–film and film–glass interfaces.

The overall low reflectance values indicate that this single-layer configuration functions primarily as an anti-reflection coating rather than as a reflector, particularly in the visible wavelength range where the reflectance remains below approximately 2%. The minimum near 550 nm suggests that the chosen thickness is close to optimal for green light. In thin-film language, destructive interference is strongest where the optical thickness produces the most nearly cancelling phase relation between the two dominant reflected amplitudes.

# 3 Double-Layer Dielectric Coating

## 3.1 Model

The bilayer structure adds a higher-index dielectric with $n_H = 2.10$ and thickness $d_H = 66\,\text{nm}$ beneath the low-index layer. The total transfer matrix is then

$$M = M_L M_H, \tag{10}$$

in the ordering used by the program.

Listing 5: Bilayer callback.

```
void total_transfer_matrix(fcomplex M[2][2], float lambda, int _) {
    const float nL = 1.38f, dL = 100.0f;
    const float nH = 2.10f, dH = 66.0f;
    const float k0 = 2 * M_PI / lambda;

    fcomplex ML[2][2], MH[2][2];
    create_transfer_matrix(ML, nL, k0, dL);
    create_transfer_matrix(MH, nH, k0, dH);
    matmult(ML, MH, M);
}
```

The reflectance spectrum for the dielectric bilayer demonstrates distinct wavelength-dependent optical behaviour across the 300 nm to 800 nm range. The curve exhibits a peak reflectance of approximately 15% centred around 420 nm, followed by a minimum of about 11% near 550 nm, after which the reflectance gradually increases towards longer wavelengths.

This profile again results from interference effects, now involving the multiple interfaces of the air–low-index–high-index–glass structure. The modest peak reflectance indicates partial constructive interference in the blue part of the visible spectrum, where the optical thickness of the bilayer more nearly approaches a quarter-wave condition. The relatively low overall reflectance of a single bilayer also explains why practical high-reflectance coatings require many repeated bilayers.

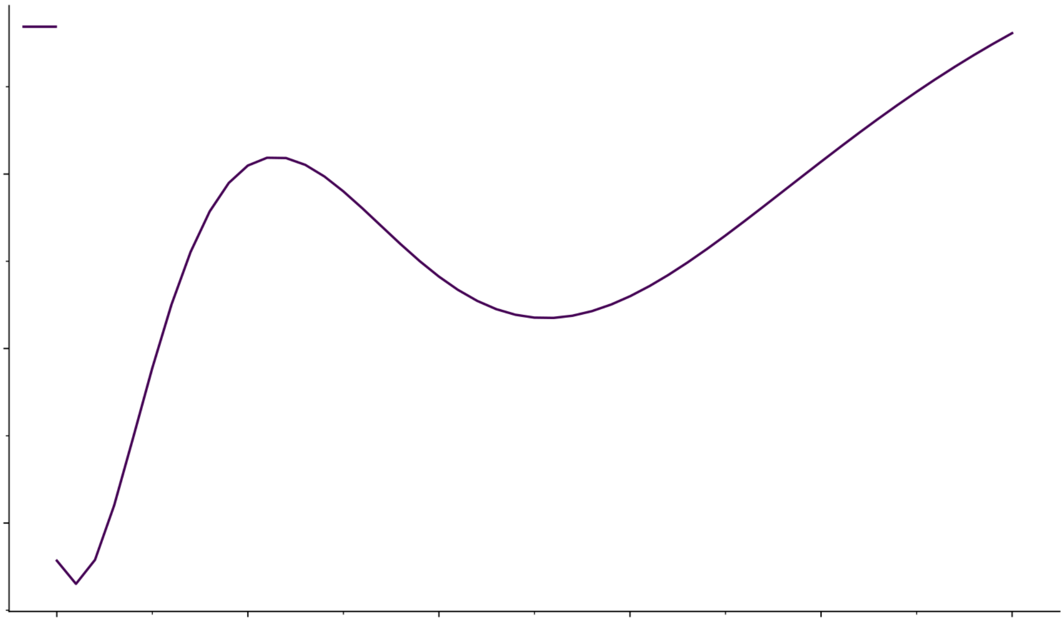

Figure 2: Reflectance spectrum of the dielectric bilayer from 300 nm to 800 nm.

# 4 Multi-Layer Dielectric Coating

## 4.1 Model

The multilayer program constructs one low/high bilayer and then repeats it. For a stack of $N$ bilayers, the total matrix is

$$M = (M_H M_L)^N, \tag{11}$$

with $N \in \{1, 3, 6, 12, 24\}$ in the present calculations.

Listing 6: Multilayer callback.

```
void total_transfer_matrix(fcomplex M[2][2], float lambda, int layers) {
    const float nL = 1.38f, dL = 100.0f;
    const float nH = 2.10f, dH = 66.0f;
    const float k0 = 2 * M_PI / lambda;

    fcomplex ML[2][2], MH[2][2], bilayer[2][2];
    create_transfer_matrix(ML, nL, k0, dL);
    create_transfer_matrix(MH, nH, k0, dH);
    matmult(MH, ML, bilayer);

    fcomplex total[2][2] = {
        { complex(1.0f, 0.0f), complex(0.0f, 0.0f) },
        { complex(0.0f, 0.0f), complex(1.0f, 0.0f) }
    };

    for (int i = 0; i < layers; i++) {
        fcomplex temp[2][2];
        matmult(total, bilayer, temp);
```

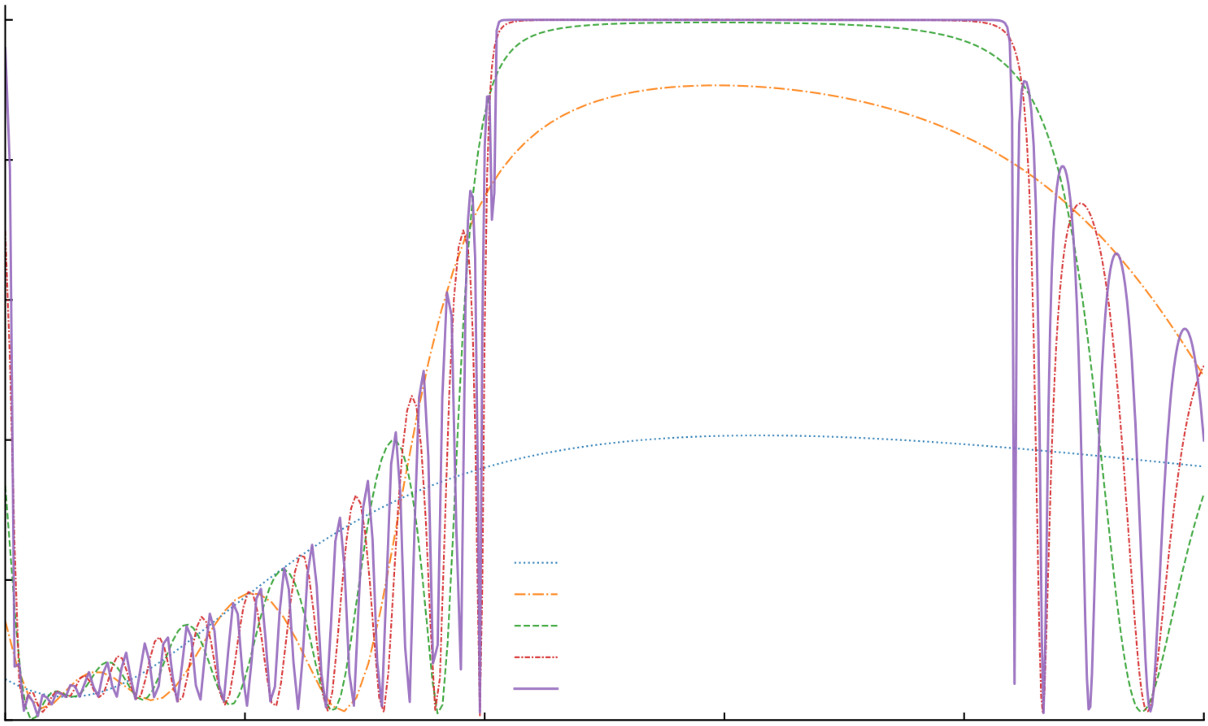

Figure 3: Reflectance spectra for 1, 3, 6, 12, and 24 dielectric bilayers.

```
        for (int row = 0; row < 2; row++)
            for (int col = 0; col < 2; col++)
                total[row][col] = temp[row][col];
    }

    for (int i = 0; i < 2; i++)
        for (int j = 0; j < 2; j++)
            M[i][j] = total[i][j];
}
```

The reflectance at 500 nm increases markedly with the number of bilayers: for a single bilayer, it remains below about 0.3, whereas for three bilayers it approaches 0.7, and for six bilayers it exceeds 0.9. By the time twelve and twenty-four bilayers are used, the reflectance near 500 nm is close to unity. Similar trends are observed across much of the visible range.

At 550 nm, a single bilayer reflects roughly 0.2 of the incident light, while three bilayers rise to approximately 0.75, and six bilayers exceed 0.9. Twelve and twenty-four bilayers maintain reflectance values close to 1.0 over a broad wavelength band, extending approximately from 480 nm to 680 nm. This progressive increase in peak reflectance, together with the broadening of the high-reflectance stop band, reflects the stronger constructive interference achieved by adding more interfaces to the multilayer stack.

It is also useful to note the oscillatory structure away from the plateau. These oscillations are the signature of alternating constructive and destructive interference among the reflected amplitudes from the many boundaries in the stack. As the number of bilayers increases, the interference becomes sharper and the stop-band mirror behaviour becomes more pronounced.

## 5 Conclusion

The reflectance characteristics of dielectric coatings follow directly from the transfer-matrix formalism and the interference of reflected waves at each interface. A single low-index layer behaves mainly as an anti-reflection coating with a shallow minimum in the visible region. A bilayer already introduces a more structured spectral response, but its reflectance remains modest. Repeating the bilayer produces the familiar behaviour of a dielectric mirror: the high-reflectance band strengthens rapidly and approaches unity as the number of bilayers increases.

Only minimal edits were required to make the original report self-contained. The main addition was the explicit inclusion of the transfer-matrix equations, the total-matrix construction, and the reflectivity formula that the code implements directly. With those definitions stated inside the document, the report now stands independently of any course handout or worksheet.